Seasonal Integration in Economic Time Series

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Abstract This paper considers various issues regarding seasonality in Australian macroeconomic time series, emphasizing the roles of unit roots and the selection of differencing filters. Many economic time series exhibit seasonal fluctuations and the nature of the seasonality, namely deterministic or stochastic, is distinguished. Although the effects of unit roots in time series have been well documented in the literature, the existence and implications of seasonal unit roots cannot be neglected in any serious econometric study. Hence, the consequences of seasonal unit roots and the importance of correct variable transformation are analysed. A brief survey of existing hypothesis tests for seasonal unit roots are presented and the HEGY test for unit roots is applied to various Australian economic time series. For certain variables, in addition to unit roots at the usual zero frequency, it is found that the hypothesis of seasonal unit roots cannot be rejected, so that the selection of appropriate differencing filters becomes paramount. In many macroeconomic time series, the commonly used first differencing filter is insufficient for the removal of seasonal unit roots, and the resultant bias in the critical values of various tests remains if seasonal integration is not considered.

1. INTRODUCTION

Seasonality has been an issue that has attracted considerable research interest in the modelling of economic time series. A broad definition of seasonality is given by Hylleberg (1992, p.4), whereby:

"Seasonality is the systematic, although not necessarily regular, intra-year movement caused by changes of the weather, the calendar, and timing of decisions, directly or indirectly through the production and consumption decisions made by the agents of the economy. These decisions are influenced by the endowments, the expectations and the preferences of the agents, and the production techniques available in the economy."

Traditionally, the approach to econometric modelling of seasonality has been to explicitly remove seasonal fluctuations from the data. The belief was that seasonality contaminates the data, adding irrelevant information to the data. Hence, a common procedure in practice has been to seasonally adjust data using techniques such as the Census X-11 procedure (see Hylleberg (1992, chapter 9)).

More recently, questions have been raised regarding the underlying validity of the seasonal adjustment approach. Many papers have argued that modelling seasonality may be beneficial for economic analysis and that the removal of seasonality may be nonsensical.

Most aggregate economic time series exhibit seasonal fluctuations. Instead of misspecifying the time series of a variable through seasonal adjustments, modelling seasonality has become a priority in its own right with the study of periodic and simple harmonic models. Consider a time series of observed total supply of fish of a particular country. It is reasonable to expect that this variable exhibits seasonal fluctuations. Naturally, the supply of fish will be higher in seasons when the weather is favourable for catches than in seasons when the weather presents difficulties. Seasonal fluctuations in this series are important as they accurately describe the variable. In this case, it is meaningless in a study of equilibrium price and quantity of seafood to consider the seasonally adjusted data. By removing seasonal fluctuations, the data are distorted. Any consequential economic studies based on the seasonally adjusted data may be spurious.

There is another argument against seasonal adjustment of time series. In the traditional Box-Jenkins approach to time series analysis, a time series is the sum of four independent components, namely a trend, cycle, seasonal and an irregular component. Seasonal adjustment mechanisms may be worthwhile only if the seasonal component is orthogonal to the other three components. Therefore, the crucial assumption underlying seasonal adjustment procedures is that it is possible to separate seasonality from a time series. It has been shown (e.g. Franses (1994)) that the assumption that the seasonal component is orthogonal to the other components of a time series is highly questionable. Hence, when seasonally adjusted data are used, in addition to the removal of a large proportion of seasonal variations, some trend and cyclical variations, which are part of the data generating process of the variable, may also be removed.

There are numerous methods for modelling seasonality in econometrics. In this paper, the focus is on seasonal integration, which is an application of the existing literature on unit root testing to seasonal time series. The purpose of this paper is to examine the nature of the seasonality that is inherent in various Australian economic time series, and to test for the presence of seasonal unit roots. In particular, whether seasonality is deterministic or stochastic, and the appropriate treatment for sensible econometric analysis, will be emphasised.

The plan of the paper is as follows. In section 2, seasonality in several Australian economic time series is explained with the aid of simple auxiliary regressions. Section 3 presents a discussion on seasonal unit roots and the importance of unit root testing. Selected economic variables are subjected to formal tests of seasonal unit roots, as well as unit roots in the zero frequency. The appropriate selection of differencing filters is addressed in section 4, which is followed by some concluding remarks in section 5.

2. SEASONALITY IN ECONOMIC TIME SERIES

It is useful to discuss various empirical regularities regarding seasonality in economic variables. The first is that seasonality constitutes a high proportion of the variation in many economic time series. Second, seasonal fluctuations do not tend to remain constant over time. Third, seasonal fluctuations are not always independent of the trend and cyclical behaviour of time series. All three of these regularities suggest that modelling seasonality is worthwhile. Franses (1994) provides a valuable discussion and presents some empirical evidence to support these regularities.

For the empirical research in this paper, quarterly data are used. It is common for analyses of seasonality to consider quarterly data because many macroeconomic data are observed quarterly. Australian data for total exports (1980Q1 to 1993Q4), total imports (1980Q1 to 1993Q4), expenditure-based Gross Domestic Product (1960Q1 to 1993Q4), retail trade turnover (1965Q1 to 1993Q4), total unemployed persons (1978Q1 to 1993Q4), and manufacturers' actual sales for clothing and footwear (1978Q1 to 1993Q4) are considered. These variables are chosen because they are important macroeconomic variables that provide an indication of the state of the economy, and also because of the relatively large amount of seasonal fluctuations they exhibit.

Many proposed functional forms of variables are non-linear. In this paper, the natural logarithmic transformation is applied to each variable. The logarithmic transformation linearises the function, allowing straightforward econometric analysis. Moreover, the logarithmic difference of data approximates the proportional growth rates of variables, which is useful for many economic interpretations.

An informal method to test for seasonality in time series is through the use of an auxiliary regression. The regression equation is of the form

$$\Delta y_t = \alpha + \delta_1 S_{1t} + \delta_2 S_{2t} + \delta_3 S_{3t} + u_t ,$$

where y_t is the variable under consideration, $\Delta y_t = (1-L)y_t = y_t - y_{t-1}$ is the first difference of y_t, L is the lag operator, α is a constant term, and S_{it} is a seasonal dummy variable that takes the value 1 in period (quarter) i and zero elsewhere. u_t is assumed to be a stationary and invertible ARMA process.

The dependent variable is the first difference of y_t, which is considered rather than the levels, in order to separate the stochastic trend component from the series. It is assumed that the first difference of y_t effectively removes the stochastic trend from the series. This assumption may be unrealistic in the absence of formal unit root testing. However, as the auxiliary regression is intended to be an informal test of seasonality, this procedure is adopted for convenience.

The regression is performed for each of the variables for their full sample as well as two subset sample sizes, each with an approximately equal number of observations. Considering split samples allows an examination of whether seasonal fluctuations change over time.

The regression results for the estimates of the parameters in the auxiliary regression, and the R^2 values, are presented in Table 1 (see appendix).

The results from the auxiliary regressions suggest that the series exhibit substantial seasonal fluctuations. The R^2 values provide an indication of the extent to which variations around the mean values of the three seasonal dummy variables affect movements of y_t around its mean. Once the trend is removed from the series, the R^2 values for the entire sample for each variable indicate that seasonality accounts from a low of 16 percent to a high of 98 percent of the variations in the variables. For the split samples of GDP, retail trade turnover and manufacturers' sales, the R^2 values all exceed 70 percent.

Diagnostic tests for spherical disturbances indicate that the residuals of the auxiliary regressions are serially correlated and, as the t-ratios will tend to be biased (with the direction of bias being dependent on the nature of the serial correlation), the estimated t-ratios are not reported. The estimated coefficients for the seasonal dummy variables for the full and sub-samples are similar. For example, the estimated coefficient for the first seasonal dummy variable for GDP for the full sample is -0.2583, and for the sub-samples they are -0.2640 and -0.2521. This suggests that, for the variables considered, the pattern of seasonality has remained fairly constant throughout the entire sample.

Based on these results, the economic variables seem to exhibit substantial seasonal fluctuations. Following this identification stage, the nature of the seasonality should be examined in more detail. Seasonality in time series can be categorised into two broad types, namely deterministic and stochastic seasonality. The econometric treatment for different types of seasonality is dependent on the nature of the seasonality.

Deterministic seasonality in time series assumes that the data generating process for the variable y_t is

$$y_t = \alpha + \beta_1 S_{1t} + \beta_1 S_{1t} + \beta_1 S_{1t} .$$

Processes such as this are purely deterministic and can be perfectly forecast. The appropriate treatment, in a regression context, for variables with deterministic seasonality is to include seasonal dummy variables in a regression model. The absence of seasonal dummy variables will lead to model misspecification and to the standard problems associated with exclusion of relevant variables.

Stochastic seasonality can be further sub-divided into stationary and integrated seasonal processes (see Hylleberg et al. (1990)). The analysis of stationary seasonal processes is standard, whereby the t-ratios are distributed according to the t-distribution. An integrated seasonal process is a process that has unit roots in the seasonal frequencies. Differencing filters are required for seasonally integrated processes. To distinguish between these two types of stochastic seasonality, formal tests must be conducted.

3. UNIT ROOT ANALYSIS

A time series is characterised by a unit root if the true value of its first lagged dependent variable is unity. Unit root processes (or I(1) processes) require first differencing of the variable for stationarity. In the absence of the appropriate variable transformation, a unit root may bias the critical values of t-tests, leading to biased inferences regarding the statistical significance of certain variables in a regression model. For valid inferences and interpretations, it is important to test for unit roots and to accommodate them accordingly.

The most commonly used test for unit roots (at the zero frequency) is the Dickey-Fuller (DF) test. The DF regression for a variable y_t is

$$\Delta y_t = \alpha + \beta y_{t-1} + u_t .$$

The null hypothesis is that y_t is non-stationary (namely, I(1) or higher), and the alternative hypothesis is that y_t is stationary, namely I(0). The test statistic is the t-ratio of the OLS estimate of β and the non-standard critical values (which change when a deterministic time trend is included in the regression) are simulated. Lagged values of the dependent variable are usually included in the regression to eliminate serial correlation in the error u_t, which leads to the Augmented Dickey-Fuller (ADF) test, based on the regression

$$\Delta y_t = \alpha + \gamma_t + \beta y_{t-1} + \sum_{j=1}^{J} \delta_j \Delta y_{t-j} + u_t ,$$

where J is the number of lags of the dependent variable. If a unit root is detected, the first differencing filter (1-L) is applied to y_t to eliminate the stochastic trend from the series. In a regression context, after differencing, regression models can be estimated by OLS and t-tests will follow the standard t-distribution.

Only recently have econometricians seriously considered the possibility and implications of seasonal unit roots. A seasonal unit root is a unit root at the seasonal frequencies of a series. With seasonal unit roots, seasonal differencing is used, namely the application of the filter $(1-L^4)$ for quarterly data and $(1-L^{12})$ for monthly data. The presence of a zero frequency unit root implies that any shock to the series has a permanent effect on the level of the series. With a seasonal unit root, a shock to the series has a permanent effect on the underlying seasonal pattern of the series.

Two issues that follow the discussion of seasonal unit roots are efficient methods for testing for seasonal unit roots and the distinction between deterministic and stochastic seasonality. Various procedures for testing for seasonal unit roots have been developed. A natural extension of the Dickey-Fuller test is the method proposed by Dickey, Hasza and Fuller (DHF). This is based on a regression of the form

$$(1 - L^s) y_t = \beta y_{t-s} + \varepsilon_t ,$$

where s=2,4,12 is the frequency of the data; for example s = 2 when semi-annual data are used. As with the ADF test, the test statistic is the t-ratio of the estimate of β. The null hypothesis is non-stationarity, and the alternative hypothesis is that y_t does not have a seasonal unit root. This regression can be augmented by lagged values of the dependent variable to eliminate serial correlation, which may bias the calculated t-statistics. The critical values are provided in DHF (1984).

There is, however, one limitation with the DHF procedure. The DHF procedure only tests for seasonal unit roots in general, and cannot distinguish between various types of seasonal unit roots, for example, annual or semiannual unit roots.

A more flexible test for seasonal unit roots is the HEGY test proposed by Hylleberg et al. (1990). This test was derived specifically for quarterly observed data but, since its introduction, an extension has been made to the monthly case by Franses (1990) and Beaulieu and Miron (1990).

Three versions of the HEGY test are considered in this paper. They differ by the number of deterministic regressors and are classified as follows:

$$a_t = \alpha + \pi_1 b_t + \pi_2 c_t + \pi_3 d_t + \pi_4 e_t + \varepsilon_t , \quad (1)$$

$$a_t = \alpha + \delta t + \pi_1 b_t + \pi_2 c_t + \pi_3 d_t + \pi_4 e_t + \varepsilon_t , \quad (2)$$

$$a_t = \alpha + \delta t + \gamma_1 S_{1t} + \gamma_2 S_{2t} + \gamma_3 S_{3t} + \pi_1 b_t + \pi_2 c_t + \pi_3 d_t + \pi_4 e_t + \varepsilon_t , \quad (3)$$

where t is a deterministic time trend, S_{1t}, S_{2t}, and S_{3t} are seasonal dummy variables, and

$a_t = (1 - L^4)y_t$,

$b_t = (1 + L + L^2 + L^3 + L^4)y_{t-1}$,

$c_t = -(1 - L + L^2 - L^3)y_{t-1}$,

$d_t = -(1-L^2)y_{t-2}$,

$e_t = -(1-L^2)y_{t-1}$, and

ε_t is an iid $(0, \sigma_\varepsilon^2)$ error term.

As with the Dickey-Fuller test, deterministic terms can be included in the HEGY test. If the error term ε_t is serially correlated, lagged values of the dependent variable (a_{t-j}, $j = 1,...,J$, where J is the lag length of the HEGY test) are included in the HEGY regression.

The first equation, (1), is the basic HEGY regression. A deterministic time trend is included in the second equation, (2), and a further three seasonal dummy variables are included in equation (3).

To motivate the null and alternative hypotheses of the HEGY test, consider the seasonal differencing filter $(1-L^4) = (1-L)(1+L)(1+L^2) = (1-L)(1+L)(1-iL)(1+iL)$, where i is a complex number. This implies that the seasonal difference operator assumes four unit roots of length 1 on the unit circle. The roots are 1, -1, i and -i, where the first root is the usual zero frequency root and the other three are seasonal unit roots. The root -1 corresponds to unit roots at 1/2 cycle per quarter or 2 cycles per year (semi-annual unit roots), and the roots i and -i correspond to unit roots at 1/4 cycle per quarter or one cycle per year (annual unit roots).

The HEGY hypotheses are formalised as follow:

1) $H_0 : \pi_1 = 0$
$H_1 : \pi_1 < 0.$

2) $H_0 : \pi_2 = 0$
$H_1 : \pi_2 < 0.$

3) $H_0 : \pi_3 = \pi_4 = 0$
$H_1 : \pi_3 \neq 0 \text{ and / or } \pi_4 \neq 0.$

The first and second hypotheses are tested using a t-test while an F test is used to test the third hypothesis. If the first hypothesis is not rejected, there is a nonseasonal or zero frequency unit root in the series. In the second HEGY hypothesis, a non-rejection of the null hypothesis indicates the presence of a semiannual unit root. Semiannual unit roots imply that any shocks to the variable will lead to permanent changes in the seasonal pattern of the variable at the semiannual level. A non-rejection of the third HEGY null hypothesis implies that the series has a unit root in the annual frequency. With annual unit roots, a shock to the variable will permanently change the seasonal pattern of the variable at the annual or yearly level. A rejection of the three null hypotheses implies that the series has no nonseasonal, semiannual and annual unit roots, respectively, for quarterly observed data.

The critical values for the HEGY test (tabulated in Hylleberg et al. 1990), like the ADF and DHF tests, are non-standard and need to be simulated. The critical values for the first two sets of hypotheses are negative. For the three HEGY tests considered in this paper, at the 5% level of significance, the t critical values range from -1.7 to -3.7, while the F critical values for the joint test range from 3.0 to 6.6. As the number of deterministic explanatory variables in the HEGY regression and the sample size both change, the critical values change.

The procedure for the testing of unit roots and the selection of the appropriate lag length undertaken in this paper is as follows. A relatively high number of lagged terms of the dependent variable is included in each HEGY regression. When diagnostic test results indicate the absence of serial correlation, lagged terms of the dependent variable are sequentially deleted to the point when the residuals are approximately white noise. The advantage of considering three HEGY equations (where (1) and (2) are nested in (3)) is that the nature of seasonality, namely deterministic or stochastic, can be considered and distinguished simultaneously. If the calculated t-values for the estimated π coefficients are markedly different between (1) and (2), it can be inferred that (1) is misspecified and hence (2) is used to test for seasonal unit roots. Similarly, the inclusion of seasonal dummy variables in (3) serves as an indirect test of whether there are any seasonal unit roots over and above deterministic seasonality in the time series. Again, if the calculated t-ratios in (3) are markedly different from those in (2), it is likely that (2) is underspecified. Beaulieu and Miron (1993) used the most general form of the HEGY regression, namely (3), and stated that seasonal dummy variables are included in their study because the loss of power from their unnecessary inclusion is insignificant compared with the bias arising from their incorrect omission.

The results of the three versions of the HEGY test, performed on the selected variables, are presented in the following tables.

Seasonal integration test results:
(Note: * indicates significant at 5%)

Total Exports:

HEGY test based on	Lags in regression	H_0: $\pi_1 = 0$	H_0: $\pi_2 = 0$	H_0: $\pi_3 = \pi_4 = 0$
(1)	8	-3.640*	-2.358*	1.545
(2)	8	-2.556	-2.419*	1.715
(3)	8	-2.468	-2.286	3.096

Total Imports:

HEGY test based on	Lags in regression	H_0: $\pi_1 = 0$	H_0: $\pi_2 = 0$	H_0: $\pi_3 = \pi_4 = 0$
(1)	8	-2.722	-0.913	4.423*
(2)	8	-2.970	-1.003	5.340*
(3)	9	-1.799	-2.391	12.652*

Nominal GDP:

HEGY test based on	Lags in regression	H_0: $\pi_1 = 0$	H_0: $\pi_2 = 0$	H_0: $\pi_3 = \pi_4 = 0$
(1)	5	-3.271*	-0.445	0.371
(2)	5	-1.827	-0.460	0.379
(3)	2	-1.469	-3.874*	17.929*

Number unemployed:

HEGY test based on	Lags in regression	H_0: $\pi_1 = 0$	H_0: $\pi_2 = 0$	H_0: $\pi_3 = \pi_4 = 0$
(1)	2	-1.799	-1.514	3.895*
(2)	2	-3.134	-1.590	3.357*
(3)	0	-2.775	-3.762*	61.035*

Retail trade turnover:

HEGY test based on	Lags in regression	H_0: $\pi_1 = 0$	H_0: $\pi_2 = 0$	H_0: $\pi_3 = \pi_4 = 0$
(1)	11	-2.657	-0.190	0.039
(2)	11	0.217	-0.186	0.035
(3)	0	0.054	-5.994*	82.417*

Manufacturers' sales (clothing and footwear):

HEGY test based on	Lags in regression	H_0: $\pi_1 = 0$	H_0: $\pi_2 = 0$	H_0: $\pi_3 = \pi_4 = 0$
(1)	1	-1.227	-2.808*	1.159
(2)	1	-2.155	-2.657*	1.108
(3)	0	-1.713	-4.552*	20.705*

The results of the HEGY tests are summarised as follows. For total exports, using (1), there is no zero frequency unit root, no semiannual unit root, but there is an annual unit root. Using (2), there is a zero frequency unit root, no semiannual unit root, and an annual unit root. The HEGY test based on (3) suggests that, in addition to the zero frequency and annual unit roots, there is also a semiannual unit root. For total imports, all three versions of the HEGY test yield consistent results, namely that imports have a zero frequency unit root, a semiannual unit root, but no annual unit root. With Australia's GDP, the results based on (1) indicate that there is no zero frequency unit root, but there are semiannual and annual unit roots. As for (2), all three unit roots exist. The test results using (3) indicate that there is a zero frequency unit root but no seasonal unit roots. The total unemployed series has consistent unit root results based on (1) and (2); there are non-seasonal and semiannual unit roots, but no annual unit root. With (3), however, the test rejected the hypothesis of semiannual unit roots. For retail trade turnover, tests based on (1) and (2) yield similar results in that there is a unit root in all the frequencies. Equation (3) indicates that there is a zero frequency unit root but no seasonal unit roots. Finally, for manufacturers' sales for clothing and footwear, the HEGY test based on (1) and (2) indicates that there is a zero frequency unit root, no semiannual unit root, and an apparent annual unit root. Test results based on (3) indicate that there is a zero frequency unit root but no seasonal unit roots.

4. DIFFERENCING FILTER SELECTION

The selection of the appropriate differencing filters should be conditional on the results of the unit root tests. However, three versions of the HEGY test were considered and, for many variables, the results of unit root testing differ with the inclusion of a time trend and seasonal dummy variables. For the analysis of differencing filters, the results of the third HEGY test (3), namely, the test which includes a constant, a time trend and three seasonal dummy variables, are considered. Except for one of the series (total imports), where all three versions of the HEGY test yield consistent results, the other variables have various types of unit roots when tested using different versions of the HEGY test. In this case, the HEGY test statistics are sensitive to the addition of deterministic terms and, hence, the omission of these deterministic terms is not sensible. Therefore, only the results from the most general version of the HEGY test is used for the determination of differencing filters.

For total exports, the appropriate differencing filter would be $(1-L^4)$, for total imports $(1-L)(1+L)$, and for GDP, total unemployed, retail trade turnover, and manufacturers' sales, $(1-L)$ would be the most appropriate filter. The results indicate that, of the six variables considered, seasonal unit roots are present in two of them, namely the two series that comprise the trade balance (total imports and exports).

As for deterministic seasonality, the different calculated HEGY statistics between HEGY regressions with and without seasonal dummy variables suggest that the series considered have deterministic seasonal components in their data generating processes. For total imports and exports, there is stochastic seasonality in addition to deterministic seasonality.

5. CONCLUSION

In this paper, several issues on seasonality in time series were discussed in connection with Australian macroeconomic time series data. Seasonal fluctuations were shown to dominate the variations in economic variables once the stochastic trend component is removed from a time series. It was also found that the underlying seasonal pattern has remained fairly constant for the entire sample considered for the variables. The importance of seasonal unit roots testing was emphasised. All variables tested have unit roots at the zero frequency. Total imports and exports in Australia are seasonally integrated as well as having the usual unit roots. For the remaining variables, namely nominal GDP, total unemployed, retail trade turnover, and manufacturers' sales (clothing and footwear), no seasonal unit roots were detected. This is consistent with the fact that the seasonal pattern of these variables has not changed over the sample. Apart from stochastic seasonality, it is likely that the variables also contain deterministic seasonal fluctuations. An application of the usual first differencing filter is inappropriate without prior testing for seasonal integration.

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APPENDIX.

Table 1: Auxiliary regression results

Variable	Sample	R^2	$\hat{\alpha}$	$\hat{\delta}_1$	$\hat{\delta}_2$	$\hat{\delta}_3$
Total Exports	82Q2-93Q4	0.0104	0.0383	-0.0131	-0.0115	-0.0279
	82Q2-87Q4	0.1623	0.0188	0.0054	0.0931	-0.0315
	88Q1-93Q4	0.3442	0.0578	-0.0318	-0.1160	-0.0243
Total Imports	82Q2-93Q4	0.2489	-0.0449	0.1463	0.0279	0.0917
	82Q2-87Q4	0.1713	-0.0377	0.1498	0.0409	0.0645
	88Q1-93Q4	0.4307	-0.0520	0.1446	0.0148	0.1189
Nominal GDP	60Q2-93Q4	0.9429	0.1231	-0.2583	-0.0705	-0.1269
	60Q2-76Q4	0.9378	0.1356	-0.2640	-0.0940	-0.1385
	77Q1-93Q4	0.9649	0.1106	-0.2521	-0.0471	-0.1152
Number unemployed	78Q2-93Q4	0.4829	0.0613	-0.0258	-0.1468	-0.0207
	78Q2-85Q4	0.4428	0.0799	-0.0661	-0.1575	-0.0528
	86Q1-93Q4	0.6030	0.0427	0.0118	-0.1361	0.0115
Retail trade turnover	65Q2-93Q4	0.9844	0.1750	-0.3374	-0.1144	-0.1580
	65Q2-78Q4	0.9818	0.1741	-0.3339	-0.1035	-0.1563
	79Q1-93Q4	0.9899	0.1758	-0.3404	-0.1246	-0.1597
Manufacturers' sales	78Q2-93Q4	0.7585	-0.0634	-0.0283	0.1399	0.2026
	78Q2-85Q4	0.8070	-0.0739	-0.0119	0.1723	0.2307
	86Q1-93Q4	0.7351	-0.0529	-0.0440	0.1074	0.1746

Note: $\hat{\alpha}$, $\hat{\delta}_1$, $\hat{\delta}_2$, and $\hat{\delta}_3$ are the OLS estimates of the constant term and the three seasonal dummy variables in the auxiliary regression. Diagnostic test results are not reported because the interest is on goodness of fit and the point estimates of the parameters.